Determination of Forward and Futures Prices

Chapter 5

Consumption vs Investment Assets

- Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)
- Consumption assets are assets held primarily for consumption (Examples: copper, oil)

Short Selling (Page 99-101)

- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way

Short Selling (continued)

- At some stage you must buy the securities back so they can be replaced in the account of the client
- You must pay dividends and other benefits the owner of the securities receives

Notation for Valuing Futures and Forward Contracts

S_0: Spot price today

F_0: Futures or forward price today

T: Time until delivery date

r: Risk-free interest rate for maturity T

1. An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a non-dividend paying stock is $40
  - The 3-month forward price is $43
  - The 3-month US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

2. Another Arbitrage Opportunity?

- Suppose that:
  - The spot price of nondividend-paying stock is $43
  - The 3-month forward price is US$39
  - The 1-year US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

The Forward Price

If the spot price of an investment asset is S_0 and the futures price for a contract deliverable in T years is F_0, then

$$F_0 = S_0 e^{rT}$$

where r is the 1-year risk-free rate of interest.

In our examples, S_0 =40, T=0.25, and r=0.05 so that

$$F_0 = 40 e^{0.05\times 0.25} = 40.50$$

If Short Sales Are Not Possible..

Formula still works for an investment asset because investors who hold the asset will sell it and buy forward contracts when the forward price is too low

When an Investment Asset Provides a Known Dollar Income (page 105, equation 5.2)

$$F_0 = (S_0 - I)e^{rT}$$

where I is the present value of the income during life of forward contract

When an Investment Asset Provides a Known Yield

(Page 107, equation 5.3)

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

Valuing a Forward Contract

Page 108

- Suppose that

  K is delivery price in a forward contract and

  F_0 is forward price that would apply to the contract today

- The value of a long forward contract, f, is

$$f = (F_0 - K)e^{-rT}$$

- Similarly, the value of a short forward contract is

$$(K - F_0)e^{-rT}$$

Forward vs Futures Prices

- Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different:
- A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
- A strong negative correlation implies the reverse

Stock Index (Page 110-112)

- Can be viewed as an investment asset paying a dividend yield
- The futures price and spot price relationship is therefore

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average dividend yield on the portfolio represented by the index during life of contract

Stock Index (continued)

- For the formula to be true it is important that the index represent an investment asset
- In other words, changes in the index must correspond to changes in the value of a tradable portfolio
- The Nikkei index viewed as a dollar number does not represent an investment asset (See Business Snapshot 5.3, page 111)

Index Arbitrage

- When $F_0 > S_0 e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures
- When $F_0 < S_0 e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index

Index Arbitrage (continued)

- Index arbitrage involves simultaneous trades in futures and many different stocks
- Very often a computer is used to generate the trades
- Occasionally simultaneous trades are not possible and the theoretical no-arbitrage relationship between F_0 and S_0 does not hold (see Business Snapshot 5.4 on page 112)

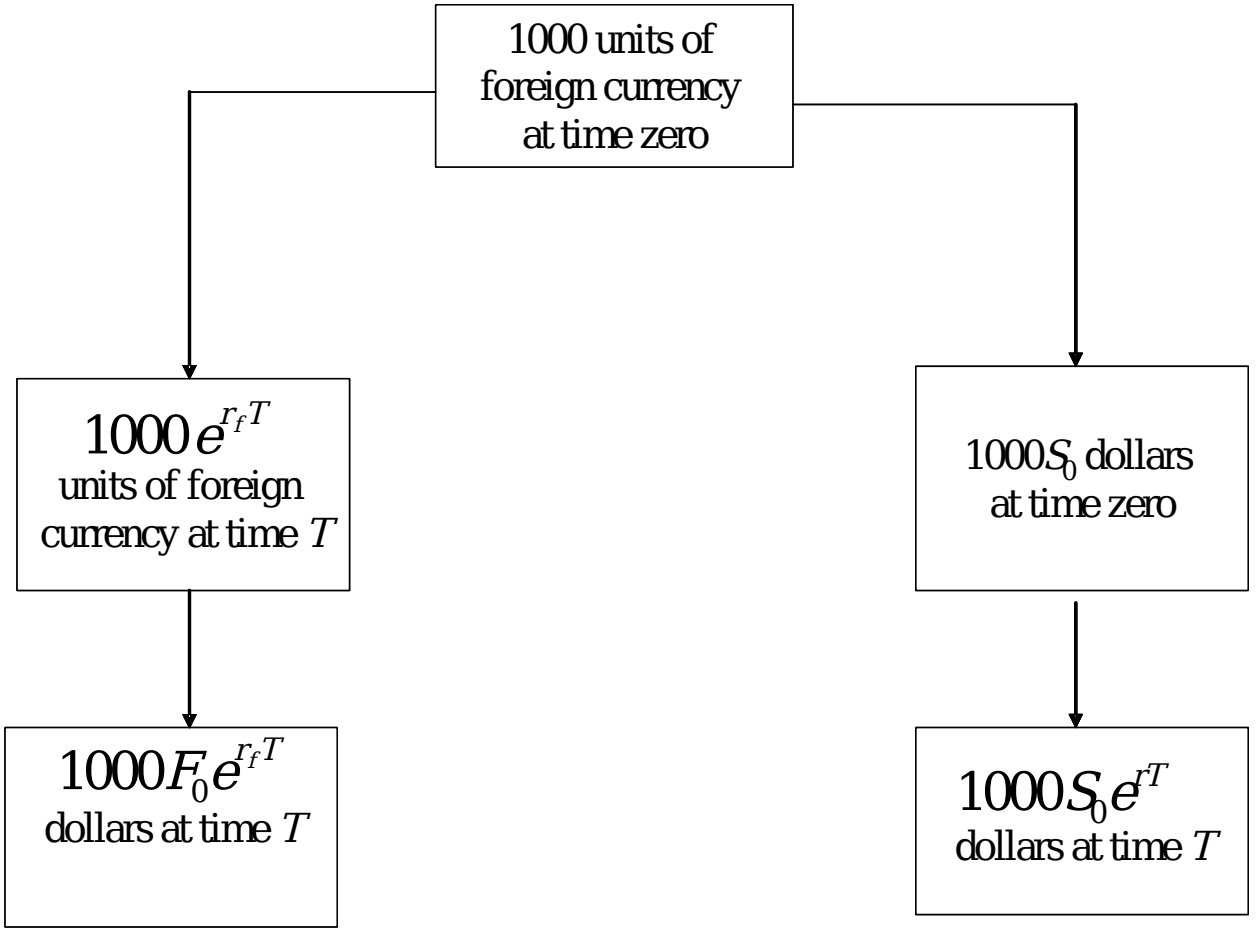
Futures and Forwards on Currencies (Page 112-115)

- A foreign currency is analogous to a security providing a dividend yield
- The continuous dividend yield is the foreign risk-free interest rate
- It follows that if r_f is the foreign risk-free interest rate

$$F_0 = S_0 e^{(r - r_f)T}$$

Why the Relation Must Be True

Figure 5.1, page 113



Futures on Consumption Assets

(Page 115-117)

$$F_0 \leq S_0 e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.

Alternatively,

$$F_0 \leq (S_0 + U)e^{rT}$$

where U is the present value of the storage costs.

The Cost of Carry (Page 118)

- The cost of carry, c, is the storage cost plus the interest costs less the income earned
- For an investment asset $F_0 = S_0 e^{cT}$
- For a consumption asset $F_0 \leq S_0 e^{cT}$
- The convenience yield on the consumption asset, y, is defined so that $F_0 = S_0\, e^{(c-y)T}$

Futures Prices & Expected Future Spot Prices (Page 119-121)

- Suppose k is the expected return required by investors on an asset
- We can invest $F_0 e^{-rT}$ at the risk-free rate and enter into a long futures contract so that there is a cash inflow of S_T at maturity
- This shows that

$$F_0 e^{-rT} e^{kT} = E(S_T)$$

or

$$F_0 = E(S_T) e^{(r-k)T}$$

Futures Prices & Future Spot Prices (continued)

- If the asset has
  - no systematic risk, then $k = r$ and F_0 is an unbiased estimate of S_T
  - positive systematic risk, then $k > r$ and $F_0 < E(S_T)$
  - negative systematic risk, then $k < r$ and $F_0 > E(S_T)$